

object of the problem, the bisection of the angle, though the line $H K$ will converge in common with the two given lines. We must further enter protest against the *unqualified* proposal "to draw a straight line equal to the true length of the circumference of a circle" (Prob. 184) as misleading to the learner. But, any such defects notwithstanding, here is a most wonderful eighteenpenny book.

LETTERS TO THE EDITOR

[The Editor does not hold himself responsible for opinions expressed by his Correspondents. No notice is taken of anonymous communications.]

Geometry at Oxford

IN the last number of NATURE Mr. Proctor remarks that "no one who considers carefully the mathematical course at either University, can believe that it tends either to form geometricians or to foster geometrical taste."

With regard to Oxford, I think it is only fair that some qualification should be offered to this conclusion. In Cambridge, candidates for mathematical honours have to run their race in a course clearly marked out for them, and loss of place is naturally the result of individual vagaries. But in Oxford the order of merit is not carried further than distribution into classes, and I do not believe there is anything to prevent a skilful geometrician finding himself in the first class with those who put their trust most in analytical methods.

I cannot pretend to much geometrical capacity, but I know something of Oxford mathematical teaching. Speaking for myself, the fascinating lectures of the present Savilian Professor of Geometry will never cease to hold perhaps the most prominent place in my recollections of university work. It is quite true that I remember conversing with a college tutor who was rather doubtful about modern geometrical methods, and seemed disposed to look upon these lectures as "dangerous." He was a great stickler, however, for "legitimacy," thinking it wrong, for example, to import differential notation into analytical geometry; but I do not think he had a large following amongst younger Oxford men. I certainly did not find, in reading with some of them, that geometry was at all in disfavour. I have often had neat geometrical solutions pointed out to me of problems where other methods proved cumbersome or uninteresting; and conversely I have found geometrical short cuts were far from objected to. On the whole, the characteristic feature of the Oxford examination system (most marked in the Natural Science School, but making itself felt in all the others) being to encourage a student after reaching a certain point in general reading to make himself strong in some particular branch of his subject, I believe special attention to geometrical methods would pay very well.

Oct. 13

W. T. THISELTON DYER

Elementary Geometry

YOUR correspondent, "A Father," has in view a very desirable object—to teach a young child geometry—but I fear that he is likely to miss altogether the path by which it may be reached. His principle, that "a child must of necessity commit to memory much that he does not comprehend," appears to me to be totally erroneous, and not entitled to be called a fact. To this time-hallowed principle it is due that a large proportion of all who go to school learn nothing at all, while those more successful learn with little improvement of their faculties. It is a convenient principle which allows the title of teacher to be assumed by those who only hear lessons. Children labour under this difficulty that they learn only through language, which is to them a misty medium, particularly when the matter set before them is in any degree novel or abstruse, and no pains are taken to clear up the obscurity of new expressions. Children know nothing of abstraction, and learn to generalise from experience, not from words. Committing to memory what is not understood is a disagreeable task; begetting a hatred of learning, and causing many to believe that they want the special faculty required for the task set before them. The art of teaching the young ought to be the art of enabling them to comprehend, and memory ought to be strengthened not by drudgery but by being founded on understanding and by the rational connection of ideas.

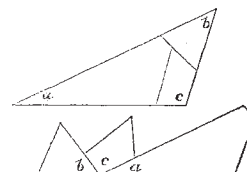
Now geometry is the science of figure; it theorises reality, and the truth of every proposition in it may be made apparent to the

senses. Double a piece of paper and cut out a triangle in duplicate. The two equal triangles thus formed, A and B , may be put together so as to form a parallelogram in three different ways. The child who makes this experiment will learn at once



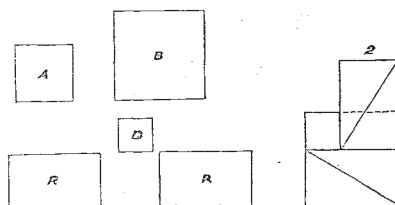
what is meant by a parallelogram, and he will perceive its properties, viz., that its opposite sides and angles are equal; that it is bisected by the diagonal, &c. But if he learns all this by rote, he acquires only a cloud of words, on which his mind never dwells. Propositions touching abstractions and generalisations can never be understood by the young without abundant illustration. When a geometrical truth is made apparent to the senses, when seen as a fact and fully understood, the language in which it is expressed having no longer a dim and flickering light, is easily learned and remembered, and the learner listens with pleasure to the discussion of the why and wherefore.

It is not enough for a child to learn by rote the definition of an angle. He ought to be shown how it is measured by a circle; and by circles of different sizes. In short, he ought to be taught what words alone will not teach him, that an angle is only the divergence of two lines. Let us now come to the important theorem that the three angles of any triangle are equal to two right angles (Euclid i. 32). Cut a paper triangle, mark the angles, then separate them by dividing the triangle and place the three angles together. They will lie together, filling one side of a right line, and thus be equal to two right angles. Let the learner test the theorem with triangles of every possible shape to convince himself of its generality, and then, fully understanding what it means, he will also understand the language in which it is proved.



It is a mistake to decry the use of symbols. They enable us to get rid of the wilderness of words, which form a great impediment in mathematical reasoning. Ordinary language can never group complex relations for comparison so compactly as to bring them within the grasp of the understanding. When we would compare objects, we place them close together, side by side. But the features and lineaments of objects described in language are too widely scattered to be kept steadily in view. It is easier to learn the use of symbols than to commit to memory what is not understood. Those who would learn mathematics without symbols can advance but a little way.

Neither is there any good reason for rejecting the second book of Euclid, though it certainly may be much abridged. The relations of whole and parts, sum and difference are easily exhibited, and an acquaintance with them is of great value to arithmeticians. Let us take for example the following propositions: "The squares (A and B) of any two lines (or numbers) are equal to double the rectangle under those lines (R and R , or the product in case of numbers) and the square of their difference D ."

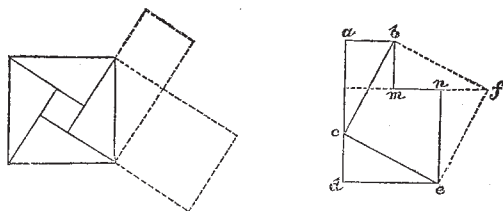


Now these figures being constructed, it will be found that when the two squares are placed together as in Fig. 2, the rectangles cover exactly the parts marked with diagonals, and the square of the difference the remainder.

In numbers, the square of 5 = 25
 " 3 = 9

Double product of 5 and 3 34
 Square of diff. 4
 34

Here is a perfect demonstration evident to the senses. But let us go one step further. The rectangles in the preceding theorem may be bisected by diagonals and set round the square of the difference in such a manner as to form the square of the hypotenuse of the right-angled triangle, the sides of which are also those of the assumed squares. The squares of the sides of a right-angled triangle, therefore, are together equal to the square of the hypotenuse, since the former may be changed into the latter.



The same conclusion may be arrived at by a still shorter and simpler course. Let any two squares be joined together as in the annexed figure, or, rather, let them be cut in paper in one piece. Then take ac equal to the side of the greater square, and join bc and ce . Cut off the two equal triangles bac and cde , and place them in the positions of bmf and gne , and the two squares will be thus transformed into the square of the hypotenuse of the right-angled triangle, of which they form the sides.

Thus we have at once a demonstration of the famous Pythagorean theorem (Euclid i. 47), and have attained with three or four steps the same height climbed by Euclid with forty-seven. The words of his demonstration, committed to memory by a child, remain there mere words and nothing more. Words serve to mark and denote ideas, but cannot create them, where the material of ideas does not already exist. But the child who with paper or card amuses himself in going over the visible demonstration suggested in this letter, in various forms and repeatedly—for neither old nor young can be said to learn a truth merely by its transient recognition—will assuredly awaken to an agreeable consciousness of the reasoning faculty, and feel no difficulty in future geometrical studies.

In 1860 there was published for me, by Messrs. Williams and Norgate, a little volume entitled, "The Elements of Geometry Simplified and Explained," adapted to the system of empirical proof, and of exhibiting the truth of theorems by means of figures cut in paper. It contained in thirty-five theorems the quintessence of Euclid's first six books, together with a supplement of thirty-three not in Euclid. There was no gap in the sequence or chain of reasoning, yet the 32nd and 47th provisions of Euclid were respectively the 3rd and 17th of my series. This book proved a failure, for which several reasons might be given, but it will be sufficient here to state but one, namely, that it came forth ten years before its time. What became of it I know not. But of this I am convinced, that though I failed, success will attend those who follow in my footsteps. W. D. COOLEY.

THE discussion in your last part on methods of teaching elementary geometry reminds me that at a period when I was teaching the subject to a considerable number of pupils, I frequently overcame the difficulties of very young or inapt students by commencing with the study of a *solid*, such as a cube, encouraging the pupils to frame definitions for the parts of the object. The ideas existing in the child's mind of a solid, a plane, a line, and a point, were thus put into words in an order the reverse of that in which they would have been had Euclid been used. The chief properties of parallelograms and triangles followed, and were easily discovered by the use of a pair of compasses, scissors, and paper, and that at an age when Euclid was a sealed book. I believe children can be most easily taught to solve problems in plane geometry when they occur in connection with early instruction in practical solid geometry. Most children try to draw, and if they were encouraged to represent simple objects by "plans" and "elevations," the necessity of obtaining a knowledge of how to describe the forms presented to them would frequently carry the pupils through a large number of the principal problems of plane geometry with a pleasure they could not experience if the "problems" were put before them, without any reason for their solution but the teacher's command. The powers of truthful representation gained by such teaching, would

be of the utmost value to thousands who would never attempt to learn "Euclid;" whilst, so far as I am able to judge, it is more likely to prepare the boy to read formal works on geometry with pleasure than to create a distaste for the study.

THOMAS JONES

Woolwich, October 9

The Coming Eclipse

I HAVE been very much interested in Mr. Lockyer's lecture at the Royal Institution on the late eclipse. I am especially glad that he is at length able to acknowledge the existence of comparatively cool hydrogen, because in my Eclipse Report of 1868 (vol. xxxvii. Part I, R.A.S. Memoirs), I stated that I believed from the evidence of the photographs that hydrogen was dispersed from the prominences in visible streams in some cases, and in others invisibly.

But while Mr. Lockyer admits this, he seems to me very unnecessarily to avoid everywhere the use of our familiar term "atmosphere" to include the whole gaseous envelope of the sun. This seems to me to be the sense in which Kirchhoff used the word when he said it was extensive.* It certainly was the sense in which I used it, and, I believe, that in which all who spoke of an extensive atmosphere did so use it. In this sense there can be no doubt that the sun has an extensive atmosphere, the outer portion of which is comparatively cold and capable of reflecting light if the polarisation now not doubted be due to reflection.

There is one consideration which, however, does not seem to have occurred to Mr. Lockyer. If the cold atmosphere, as I will venture still to call it, reflect the prominence light, it will also reflect the solar light. Its reflected light then should be such as reaches us at ordinary times, and not so exclusively chromospheric. Adding to this the light which is due to cool hydrogen, we should have, I anticipate, a faint continuous spectrum with the bright line F, and also a solar spectrum with, perhaps, some of the chromospheric lines reversed. That is not what has been found, and I do not at present see any way of reconciling the facts with the theory that the undoubted polarisation is due to reflection.

Before going to another subject, I would wish to direct attention to my friend Captain (now Major) Branfill's observations in 1868† on the polarisation of the corona. Mr. Proctor, indeed, in his book on the Sun, says that the Astronomer Royal did not consider them conclusive, but I have his official statement that he did so consider them, and an inquiry as to Mr. Proctor's authority leads me to think that Mr. Airy's meaning was mistaken. I think any one who reads the account in the original will feel that the plane of polarisation was satisfactorily determined. An observer in 1870 has said that he found the bands of Savart persistent. I have not now time to look up the reference, but he used, it seemed to me, the centre of the moon as the centre of rotation. Captain Branfill was careful not to do this, as his figures prove (page 25 of Report).

Now to the future. I have received from Government an inquiry as to recommendations to observers coming out. I am now suggesting, in addition to my own station at Dodabetta, that observers should be stationed at Kotagherry in the Nilgherries, at Manantoddy among the coffee districts to the west, and at Tirupur, close to Avenashy Road Station of the Madras Railway. Of these Manantoddy is the least accessible, but the whole will give a range of stations from 8,600 feet high down to the ordinary level of the plain country. More observers could be accommodated on the Nilgherries, where the weather, I am assured, is likely to be excellent. Of Ceylon I have not satisfactory accounts, nor of the west coast.

If these stations be adopted, I would suggest that, if possible, there should be a conference of observers. The possibility will depend on our leisure, which, probably, none of us can now foresee.

I should say that I have made these suggestions without reference to Mr. Pogson, because I know nothing of his plans, having received no answer to inquiries; it is possible those may modify projects, but any visitors should bear in mind that it is almost necessary that some European residences should be close to their stations.

J. T. TENNANT

H.M. Mint, Calcutta, Sept. 11

* Mr. Lockyer has long ago shown that the Sun's atmosphere lies partly above and partly below the superior limit of the photosphere. The word Atmosphere was used by Kirchhoff in the manner indicated, because he believed the photosphere to be liquid.—Ed.

† American observers seem never to have seen the Report.